Sections 2.1 and 2.2

Concept of a Limit

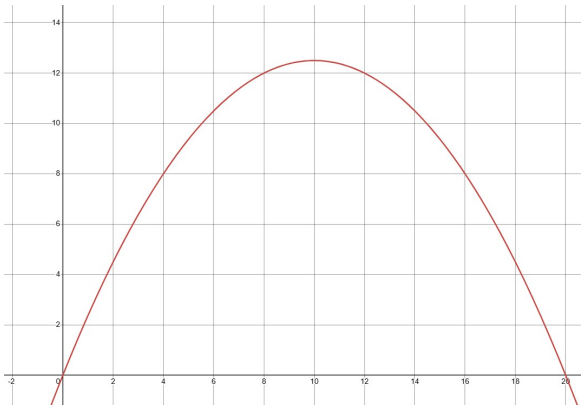
We begin with a motivating example from physics: velocity.
If we travel in our car 30 miles in half an hour, our *average velocity* is:

But at a given moment our speedometer is unlikely to read exactly.
The speedometer reading is the *instantaneous velocity.*

Example 2.1.1. Suppose a ball is tossed in the air and the height is measured at time t by the function

$$h(t) = -\frac{1}{8}t^2 + 2.5t$$

Follow Desmos graph 1 and Desmos graph 2 for interactive graphs of h.



Find the average velocity of the ball from times $t = 2$ to $t = 10$. What does this quantity represent graphically?

How can we find the instantaneous velocity of the ball at time $t = 2$.

This construction of the instantaneous velocity using average velocities is our prototype for the concept of limits.

Graphically this same calculation showed the limit of the slopes of the secant lines was the slope of the tangent line. For general functions we call these the *average rate of change* and *instantaneous rate of change.* The formula is:

Example 2.1.2. Find the tangent line to $f(x) = 15 - 2x^2$ at $x = 1$.

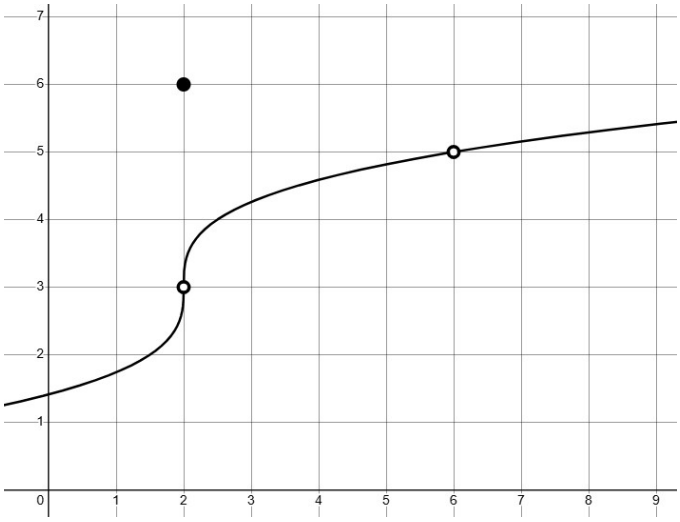
The Definition of a Limit

Definition 2.2.3 (Limit of a Function)**.** If a function f is defined for all x values close to c (possibly undefined at c) and if $f(x)$ gets as close as we'd like to L for x as close as we need (but not equal) to c then we write

$$\lim_{x \to c} f(x) = L$$

Read this as:

Example 2.2.4. We can use graphs to determine limits:



1. Determine $f(2)$ and $\lim_{x \to 2} f(x)$

2. Determine $f(6)$ and $\lim_{x \to 6} f(x)$

Until we get better methods of computing limits, we prefer using graphs over tables of values. Why?

Example 2.2.5. Plug in values near 0 to determine the following limit (Desmos graph)

$$\lim_{x \to 0} e^{-(10^{10}x)^2}$$

Definition 2.2.6 (One-Sided Limits)**.** Let f be some function.

1. **Right-sided limit** If f is defined near c for all $x > c$ and $f(x)$ is arbitrarily close to L for x sufficiently close to c with $x > c$ then we write

$$\lim_{x \to c^+} f(x) = L$$

2. **Left-sided limit** If f is defined near c for all $x < c$ and $f(x)$ is arbitrarily close to L for x sufficiently close to c with $x < c$ then we write

$$\lim_{x \to c^-} f(x) = L$$

Why might we want to define these other types of limits?

Theorem 2.2.7. Suppose f is defined for all x near a value c (possibly not defined at $x = c$). Then $\lim_{x \to c} f(x) = L$ if and only if

$$\lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)$$

Example 2.2.8. Evaluate the left, right, and two-sided limits of $g(x) = \dfrac{2x^2 - 6x + 4}{|x - 1|}$ as x approaches 1.